Assignment Ultrasound-Imaging

The purpose of the assignment is to provide an analysis for the selection of the dimensions for the elements of a ultrasound transducer. In order to do so, the directivity profile from three different possible widths is studied and they steering capabilities, in a phased array configuration, is discussed. In addition, a suggestion for avoidance of grating lobes and a recommendation in the width selection for a particular application is provided.

Design Constraints

The desired ultrasound probe must work with the following characteristics:

- **Operation Frequency:** $3.5MHz$
- **Number of Elements:** 64
- **Height of Elements:** $15mm$
- **Space between Elements:** *Negligible*

The elements width for the transducer elements can be chosen from

- **Available Widths:** $0.1mm$ $0.2mm$ $0.3mm$

where $0.3mm$ is favored by the chief designer.

Directivity Analysis

Using the Fraunhofer approximation for far field,

$$D(\theta) = sinc(W\frac{\sin(\theta)}{\lambda})$$

the directivity for each of the different widths (azimuthal profile) is evaluated.

For this and further analysis, we use a velocity for soft tissue given by:

$$c = 1540\, m/s$$

which gives a wavelength

$$\lambda \approx 440\mu m$$

.

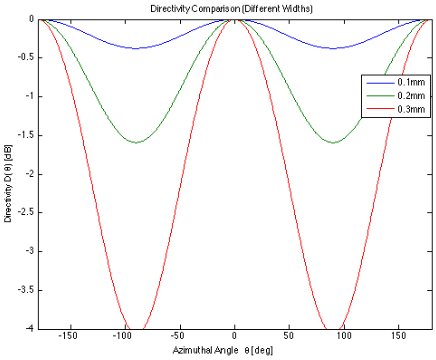

Figure 1: Comparison of Directivity between different element widths

From Fig. 1 is seen that as the ratio $\frac{W}{\lambda}$ becomes larger, the power is concentrated better at the origin ($\theta = 0^o$). As this ratio becomes smaller, an almost constant profile is observed.

In addition, as all possible configurations share the same height a unique elevation profile is shown in Fig. 2. These directivity profiles are computed by considering both directions independently.

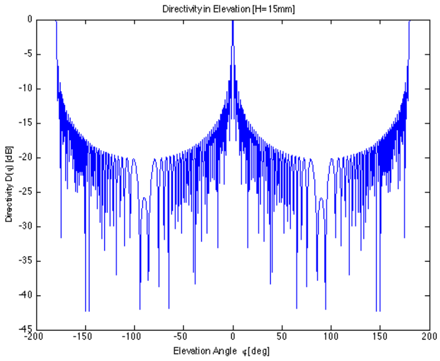

Figure 2: Elevation Directivity Profile

As expected the elevation profile is concentrated at $\phi = 0^o$ (large ratio between height and wavelength), showing a well defined peak.

Directivity of Phased Array

As phased array transducers are ruled by the geometry of the array and the size of the individual element, the following analysis generates the directivity profiles for a phased array, with different steering angles, constructed using sets of components of the available widths. This will show the characteristic of the virtual aperture created by the phased array.

Figures below show the steering capabilities for each of the arrays in 5^o steps.

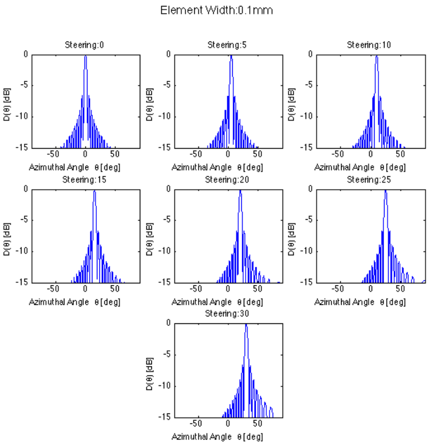

Figure 3: Phased Array Directivity for $0.1mm$ elements

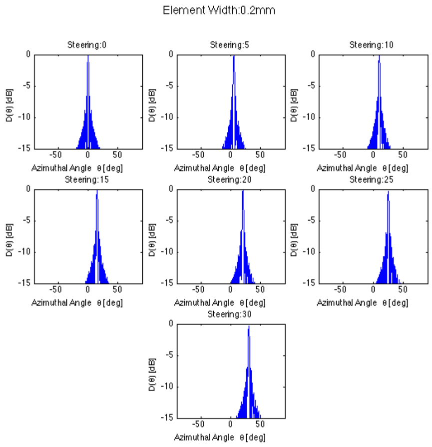

Figure 4: Phased Array Directivity for 0.2mm elements

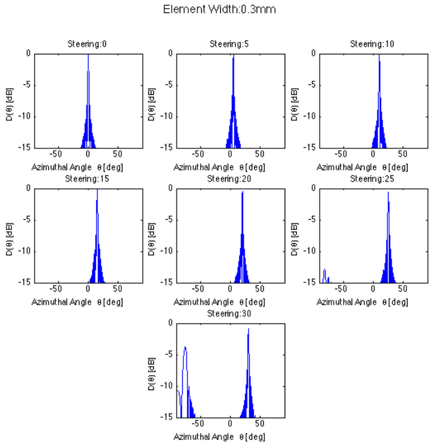

Figure 5: Phased Array Directivity for 0.3mm elements

From the previous figures it is observed two main effects: main lobe attenuation (decrease in the power of the steered beam) and granting lobes appearances.

The first effect is results from the natural beam spreading for each element in the array. From the theory it can be recalled that the maximum spreading angle ($@-6dB$) is given by

$$\sin\left(\theta_{st}\right)=0.514\frac{\lambda}{w_e}$$

where
θ_{st} is the maximum steering angle.
λ is the wavelength.
w_e is the element width.

The relation implies that as the width of the elements is reduced higher angular energy content is present, which can be combined in order to maximize steering, at the cost of some losses in the power of the main beam.

On the other hand, as seen in Fig. 5, the existence of granting lobe can be a problem if a high steering angle is needed. In order to detect the maximum steering angle, which keep grating lobes under $6dB$ with respect the main beam, a trial-and-error approached was used.

Instead of plotting all the profiles again and search for the angle manually, the power of the spectra was computed and sorted in decreasing order, expecting to find two *cumulative* peaks (CP) in the graph (one corresponding to main beam and the other for the grating lobe). The CP are compared with the $-6db$ threshold and the first incidence is considered as the maximum steering angle.

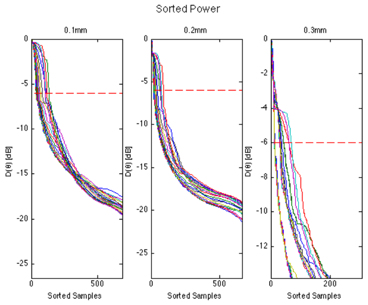

Figure 6: Sorted Spectra samples for detecting the CPs

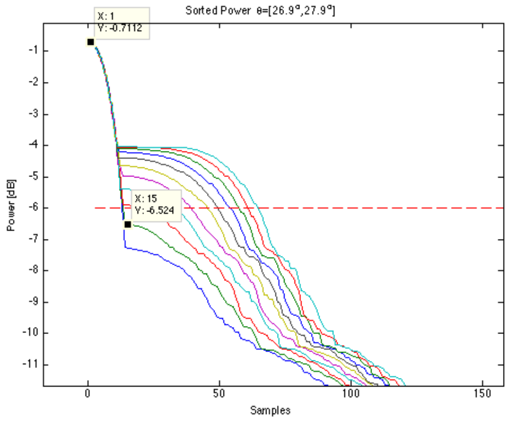

Figure 7: Sorted Spectra samples for angles $\theta=[26.9,27.9]$.

From Fig. 6 it seen that no grating lobe appears with elements width 0.1mm and 0.2mm. For 0.3mm the grating lobe is observed from angles around 25^o degrees. Using the information from the plot in Fig. 7, the empirical angle was found to be around 27^o. Fig. 8 show the evolution of the grating lobe for element width of 0.3mm.

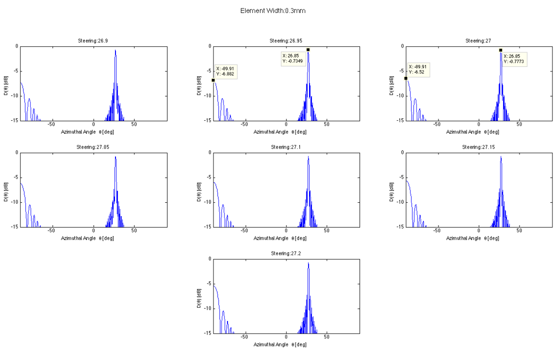

Figure 8: Directivity Profile with Grating Lobe for 0.3mm width.

As the amplitude of grating lobes is significantly affected by pitch size (distance between elements), the number of elements, frequency, and bandwidth, good practices in design and current available market technology should be taken into account to deal with this problem.

In general, for phased arrays, grating lobes will appear whenever the size of individual elements surpass a wavelength, and there will be no grating lobes when element size is smaller than half of it. In case we found ourselves in between this range, grating lobes are function of the steering angle as shown before.

The straightforward solution is the use of a smaller pitch, but in this case we assume that the distance between element is negligible, hence if the application needs steering capabilities over 27^o degrees the elements width should be reduced.

Conclusions and Recommendations

As seen in the analysis, higher pitch, frequencies and element widths hinders the steering capabilities and causes granting lobes at small steering angles. However, even if it is desired a smaller element the constraints in the manufacturing process has to be also considered.

Considering a fair trade-off between performance and available technology, the recommended element width for the transducer will be of 0.2mm. This width represents a balance between current market supply [3], width original goal (0.3mm) and good performance (based in directivity analysis). This can be considered a common design choice for a sector scan, which under the design specification, can be applied in heart imaging.

References

1. Epstein, Charles L.. **Introduction to the Mathematics of Medical Imaging**. 2003 Prentine Hall
2. Arnulf Oppelt. **Imaging Systems for Medical Diagnostics: Fundamentals, Technical Solutions and Applications for Systems Applying Ionizing Radiation, Nuclear Magnetic Resonance and Ultrasound**. John Wiley & Sons, 2011
3. OLYMPUS **Phased Array Tutorial** http://www.olympus-ims.com

Matlab Code

```
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Medical Imaging
% Ultrasound Assignment
%
% Design of Ultrasound Transducer
% Mario Coutino
% TU Delft 2015
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all, close all

% Design Variables
freq = 3.5e6;                % Op. Freq.
numEl = 64;                  % Num. of Elements
H = 15e-3;                   % Height of Elements
W = 1e-3*[0.1 0.2 0.3]; % Width of Elements

softTissueVel = 1540;    % Given vel. in soft tissue [m/s]

lambda = softTissueVel/freq; % Wavelenght inside soft tissue

% Directivity Function in Fraunhofer Approx (Far-Field)
theta = linspace(-pi,pi,1e3);
Wx = repmat(W',1,length(theta));
St = repmat(theta,length(W),1);
Dx = sinc(Wx.*sin(St)/lambda); % Multiple Widths in Dx (Azimuth)
Dy = sinc(H*sin(theta)/lambda); % Single Height in Dy (Elevation)

% dB Scales
DxdB = 10*log10(abs(Dx));
DydB = 10*log10(abs(Dy));

```

```
plot(rad2deg(theta),DxdB')
xlabel 'Azimuthal Angle \theta [deg]';
ylabel 'Directivity D(\theta) [dB]';
title 'Directivity Comparison (Different Widths)'
L = {'0.1mm','0.2mm','0.3mm'};
legend(L);
axis([-180 180 -4 0])

figure, plot(rad2deg(theta),DydB)        % This Plot seems weird to me...
xlabel 'Elevation Angle \phi [deg]';
ylabel 'Directivity D(\phi) [dB]';
title 'Directivity in Elevation [H=15mm]'

%% Phase Array

d = W;                       % The distance between elements is neglected
k = 2*pi/lambda;             % Wave number
% ang = 0:5:30;
ang = 25:0.5:28
steerAngle = deg2rad(repmat([25:.5:28]',1,length(theta)));

sTheta = repmat(theta,size(steerAngle,1),1);

for i = 1:length(d)
    arrayGain(i,:,:) = ( sin(0.5*numEl*( k*d(i)*(sin(sTheta) - sin(steerAngle)) ) ) )' ./...
            ( numEl*sin( 0.5*( k*d(i)*(sin(sTheta) - sin(steerAngle)) ) ) )';
end

for i = 1:size(arrayGain,3)
    arrayPower(:,:,i) = arrayGain(:,:,i) .* Dx;
end

for el = 1:length(d)
    figure,
    for i = 1:size(steerAngle,1)
        j = i;
        if i == size(steerAngle,1)
            j = j + 1;
        end
        subplot(3,3,j)
        aPowerdB = 10*log10(abs(arrayPower));
        plot(rad2deg(theta),aPowerdB(el,:,i))
        xlabel 'Azimuthal Angle \theta [deg]'
        ylabel 'D(\theta) [dB]'
        str = strcat('Steering: ', num2str(ang(i)));
        title(str)
        ylim([-15 0])
        xlim([-90 90])
    end
    str = strcat('Element Width: ',L{el});
    suptitle(str)
end

%%
% Maximum Steering Angle (Grating Lobe below -6dB)
% Elements spacing below half lambda assure no grating lobes
maxAngle = (asin(0.514*lambda./W));
angle = [25:.1:30];
% steerAngle = deg2rad(repmat([0:.1:28]',1,length(theta)));
steerAngle = deg2rad(repmat([0:5:90]',1,length(theta)));

sTheta = repmat(theta,size(steerAngle,1),1);
clear arrayGainT
clear arrayPowerT
% clear L1
clear indx
for i = 1:length(d)
    arrayGainT(i,:,:) = ( sin(0.5*numEl*( k*d(i)*(sin(sTheta) - sin(steerAngle)) ) ) )' ./...
            ( numEl*sin( 0.5*( k*d(i)*(sin(sTheta) - sin(steerAngle)) ) ) )';
end

for i = 1:size(arrayGainT,3)
    arrayPowerT(:,:,i) = arrayGainT(:,:,i) .* Dx;
end

for i = 1:size(steerAngle,1)
    aPowerdB = 10*log10(abs(arrayPowerT));
    [L1(:,i) indx(:,i)] = sort(aPowerdB(1,:,i),'descend');
end
% figure,
% plot(repmat(rad2deg(theta)',1,11),L)
subplot(1,3,1)
plot(L1);
hold on, plot(1:1e3,-6*ones(1e3,1),'--r')
title('0.1mm')
```

```
xlabel 'Sorted Samples'
ylabel 'D(\theta) [dB]'
subplot(1,3,2)
plot(L2);
hold on, plot(1:1e3,-6*ones(1e3,1),'--r')
title('0.2mm')
xlabel 'Sorted Samples'
ylabel 'D(\theta) [dB]'
subplot(1,3,3)
plot(L3);
hold on, plot(1:1e3,-6*ones(1e3,1),'--r')
title('0.3mm')
xlabel 'Sorted Samples'
ylabel 'D(\theta) [dB]'
suptitle('Sorted Power')
```